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THE CONSTANT OF REFRACTION.

By Russell Tracy Crawford.

(Read before the meeting of the Society held in Berkeley, January 30, 1904.)

The phenomenon known as refraction was first mentioned by and probably discovered by Ptolemy, who lived about the second century A. D. In his famous treatise on optics he points out that the light coming to us from a star or heavenly body, on entering our atmosphere and traversing it to its lower and denser portions, is gradually bent or *refracted,* the result being that the object appears to the observer to be nearer the zenith than it actually is. He shows further that this bending ranges from zero at the zenith to a maximum at the horizon.

Walther, who worked in the fifteenth century A. D., was the first one to make any successful attempt to allow for atmospheric refraction in reducing observations.

Tycho, who is celebrated for his wonderful series of accurate observations, recognized fully the importance of refraction, and consequently made a series of observations to find out the amount of displacement of an object, due to this bending for different parts of the sky, and constructed the first table of refraction. Although it was not a very accurate one, it marks an epoch in the study of this phenomenon. Kepler, following Tycho, made a considerable improvement in the theory of astronomical refraction.

The law of refraction was discovered by Snell in the early part of the seventeenth century. In the latter part of this same century, Cassini computed a new table of refractions which was an improvement upon Kepler's. This was, in turn, followed by a further improvement by Bradley in the early part of the eighteenth century.

About 1818 Bessel gave us his theory of refraction, which is the one used to-day. "Although the complete theoretical solution was then, as now, unattainable, he succeeded in constructing a table of refractions which agreed very closely with observations, and was presented in such a form that the necessary correction for a star in almost any position could be obtained with very little trouble."

The importance of refraction in any problem involving, either directly or indirectly, the zenith distance of an object is evident. The amount of the bending depends upon two things, the density of the air surrounding the observer, and the angle at which the ray of light strikes the refracting medium. As for the variation with the angle of incidence, we may say that refraction varies (very nearly) directly as the tangent of the angle of incidence. The density of the air depends principally upon the elevation above sea-level, and the temperature. BESSEL's theory takes account of all these things, and in this theory is involved what is known as the constant of refraction.

We might now ask what we mean by the constant of refraction, or what is the constant of refraction. It is *not* a number representing something of which we can form a definite picture, as we can of the constant of aberration, for instance, which is the angle whose tangent is the ratio of the velocity of the Earth in its orbit to the velocity of light; but it is one of two numbers which enter as constants into the general expression of refraction, and cannot be described in words, but merely represented analytically. These two constants are usually designated α and β; but the one called α is known as *the* constant of refraction.

The values of these constants may be found from theory by making some assumption as to the law of the decrease of density of the air for increasing heights above the surface of the Earth. Several such assumptions have been made and tables of refraction deduced from them; some of them give values of the refraction which agree fairly well with observations for a short distance from the zenith, but fail utterly for large zenith distances. The hypothesis presented by BESSEL, however, holds very well indeed for all zenith distances down to about 85°. It is upon this hypothesis that modern refraction tables are based.

First, then, tables were made from these theoretical values of the constants, and corrections have since been made to them by making use of the discrepancies found to exist between the theoretical values of the refractions and the actually observed values. These discrepancies are small; so small, in fact, that we may regard the formula for refraction, which involves these constants, as representing the *law of refraction* well enough, but they are too large to give very fine results, such as are needed in work with the meridian-circle. The constants of refraction, therefore, must receive small corrections in order to make theory and observation harmonize. The constant β is small, and is undoubtedly very accurately known, so that our attention, for the present, will be directed wholly to *the* constant of refraction, α.

Considering, then, that no correction to β is needed, the following equation can be shown to exist:—

$$dr = \frac{r}{\alpha} d\alpha$$

in which r is the refraction, α the constant, dr the difference between the observed and the computed refraction, and $d\alpha$ the small correction to be applied to the constant, upon which the computed refraction depends, to obtain the constant which will agree with the observations.

The whole problem, therefore, resolves itself into finding some means of deriving the true refractions from observations so that we may form the quantity dr. There are several methods of obtaining refractions from observations. The one generally used, and which gives excellent results, is by observations of circumpolar stars. I shall explain this method in the words of YOUNG, taken from his General Astronomy:—

> "At an observatory whose latitude exceeds 45° select some star which passes through the zenith at upper culmination. It will not be affected by refraction at the zenith, while at the lower culmination, twelve hours later, it will. With the meridian-circle observe its polar distance in both positions, determining the 'polar point' of the circle in the usual way. If the polar point were not itself affected by refraction, the simple difference between the two results for the star's polar distance, obtained from the upper and lower observations, would be the refraction for the lower point.
>
> "As a *first approximation,* however, we may neglect the refraction at the pole, and thus obtain a *first* approximate lower refraction. By means of this we may compute an *approximate* polar refraction, and so get a first 'corrected polar point.' With this compute a second approximate lower refraction, which will be much more nearly right than the first; this will give a second 'corrected polar point'; this will in turn give us a third approximation to the refraction; and so on. But it would never be necessary to go beyond the third, as the approximation is very rapid. If the star does not go exactly through the zenith, it is only necessary to compute each time approximate refractions for its upper observation, as well as for the polar point."

This method, as I have said, gives excellent results, but it has several disadvantages, which it might be well to note. In the first place, it is easily seen that the computations required are quite complicated because of the approximations which have to be made; further, the latitude and its variation are involved; again, the field of observation is limited; and finally, there must be a wait of twelve hours or six months between the observations at upper and lower culminations. If this wait is twelve hours, one observation, in general, will be made in the daytime under entirely different atmospheric conditions from the one made at night; if the wait is six months, so that both observations may be made at night, the atmospheric conditions will again probably be entirely different at the times of the two observations.

It is my purpose now to present briefly another method by which refractions may be observed. It may be stated as being a "quasi" converse to TALCOTT's method of determining latitude. Instead of eliminating refractions to get the latitude the method is to determine the refractions by eliminating the latitude, as follows:—

Let z_s = the zenith distance of a southern star.
z_n = the zenith distance of a northern star.
z'_s = the apparent zenith distance of the southern star.
z'_n = the apparent zenith distance of the northern star.
δ_s = the declination of the southern star.
δ_n = the declination of the northern star.
r_s = the refraction of the southern star.
r_n = the refraction of the northern star.
ϕ = the latitude of the meridian-circle.

Then
$$\delta_n = \phi + z_n = \phi + (z'_n + r_n) \tag{1}$$
$$\delta_s = \phi - z_s = \phi - (z'_s + r_s) \tag{2}$$
$$\delta_n - \delta_s = z'_s + z'_n + r_s + r_n \tag{3}$$

Let
$$A = \delta_n - \delta_s \tag{4}$$
$$B = z'_s + z'_n \tag{5}$$

Then
$$A = B + r_s + r_n \tag{6}$$

or
$$r_s + r_n = A - B \tag{7}$$

If now the southern and northern zenith distances were the same, and if, at the times of observing them, the conditions of the atmosphere were the same, the two refractions would be the same,—i. e.—

$$r_s = r_n$$

In this case we have

$$2r = A - B \tag{I}$$

In practice these ideal conditions are only approximately satisfied. We therefore proceed as follows:—

From (7) we have

$$2r_s - r_s + r_n = A - B \tag{8}$$

whence

$$2r_s = (A - B) + (r_s - r_n)$$

and, also

$$\left.\begin{aligned} r_s &= \tfrac{1}{2}(A - B) + \tfrac{1}{2}(r_s - r_n) \\ r_n &= \tfrac{1}{2}(A - B) + \tfrac{1}{2}(r_n - r_s) \end{aligned}\right\} \tag{II}$$

In case the northern star is at lower culmination we shall have:

$$\delta_n = 180^\circ - z_n - \phi \tag{9}$$

$$\delta_s = \phi - z_s \tag{10}$$

$$\delta_n + \delta_s = 180^\circ - z_n - z_s \tag{11}$$

$$= 180^\circ - [z'_n + r_n + z'_s + r_s] \tag{12}$$

Hence

$$r_n + r_s = 180^\circ - [z'_n + z'_s] - [\delta_n + \delta_s] \tag{13}$$

and

$$2r_s = 180^\circ - [z'_n + z'_s] - [\delta_n + \delta_s] + [r_s - r_n] \tag{14}$$

Calling

$$A' = \delta_n + \delta_s \tag{15}$$

and since

$$B = z'_s + z'_n \tag{5}$$

we have and

$$\left.\begin{aligned} r_s &= 90^\circ - \tfrac{1}{2}[A' + B] + \tfrac{1}{2}[r_s - r_n] \\ r_n &= 90^\circ - \tfrac{1}{2}[A' + B] + \tfrac{1}{2}[r_n - r_s] \end{aligned}\right\} \tag{III}$$

In order to obtain the refractions from (II) and (III) it is necessary to know the declinations of the stars, their apparent zenith distances (or, rather, the sums of the zenith distances of the pairs of north and south stars), and the differences between the refractions of the pairs.

Observations were made at the Lick Observatory by myself, in accordance with this method during a few months of 1899. The stars used were all fundamental, and in a first approximation their declinations are to be considered absolute. They were taken from Professor NEWCOMB's "Catalogue of

Fundamental Stars for 1875 and 1900, Reduced to an Absolute System." The apparent zenith distances, or the sums of the zenith distances of the several pairs, are obtained from the meridian-circle observations; and the differences in the refractions are found by computing the refractions from some standard table. In this work the Pulkowa tables have been used. The term ½ $(r_s - r_n)$ being of the nature of a differential refraction, any error in the constant of refraction of the table used will have practically no effect upon this difference. The more nearly ideal conditions (i. e. when $r_s = r_n$) are approached, of course, the better the determination of the refractions will be.

This method has both its advantages and its disadvantages. Among the former, the most important are: First, the total elimination of the latitude, and hence also of its variation; second, the elimination of the nadir, since $(z'_s + z'_n)$ is nothing more nor less than the difference between the circle-readings, and is therefore independent of the zenith point; third, there is no wait of twelve hours or of six months in order to observe a star at both culminations, as is usually done; and fourth, the simplicity of the reductions.

The greatest disadvantage in this method lies in the fact that the declinations of the stars have to be considered known. But by taking fundamental stars, such as those whose places are given by Professor NEWCOMB's new Fundamental Catalogue, and by taking a large number of these stars, this difficulty will be nearly completely eliminated.

The value of the constant found from these observations, reduced by the method just given, is somewhat smaller than that used in the Pulkowa refraction tables. It is 60″.159 for a pressure of 760^{mm} at 0° and 0° (C.) temperature.

For the sake of comparison, the most important determinations of the constant of refraction are given below. These values are for the conditions $B = 760^{mm}$ at 0° C. and external thermometer = 0° C. (These values are taken from Professor BAUSCHINGER's "Untersuchungen über die Astronomische Refraction u. s. w.").

1. Fund. Astr.	60″.320
2. Tab. Reg.	.440
3. Tab. Pulk.	.268

4. Fuss.	.122
5. Greenw. 1857-1865	.120
6. Pulk. 1865	.209
7. Greenw. 1877-1886	.192
8. Pulk. 1885	.058
9. München	.104

The first and second of these are determinations by BESSEL; the third by GYLDÉN; the fifth, by STONE; the sixth, by NYRÉN; the seventh, by NEWCOMB; the eighth, by NYRÉN; and the last, by BAUSCHINGER.

BAUSCHINGER gives weight zero to each of BESSEL'S determinations; to the first because there was considerable uncertainty in BRADLEY'S meteorological instruments; to the second, because of the uncertainty in reading the meridian-circle (read by vernier to one second). He gives equal weight to the last seven, and gets for a mean

$$\alpha = 60''.153$$

The *apparently* fine agreement of my value with this mean, however, is purely accidental, as can be seen from some further reductions of the observations which were made later. It was noticed that the value of $d\alpha$ derived from individual pairs of stars was a function of the zenith distance of the pair used. Upon plotting these values, using the zenith distance z for abscissa, and $d \log \alpha$ for ordinate, it was easily seen that these values varied quite uniformly with the zenith distance. Therefore, a term depending upon the zenith distance was introduced and the resulting equations solved by the method of least squares. This solution led to the conclusion that the so-called constant of refraction used in the Pulkowa tables needs not only a correction, but a correction for every zenith distance. In other words, the formula from which refractions are computed needs to be modified. Or the formula may be retained unaltered, and the desired agreement between observation and computation may be obtained by correcting the tables used (Pulkowa) not by a constant amount but by a variable one represented in magnitude by the expression

$$\Delta \log \alpha = 0.000101 \ [56^\circ\ 38'.8 - z]$$

where z is the zenith distance.